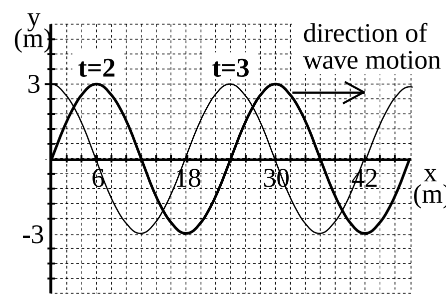
MULTIPLE CHOICE SOLUTIONS--MECHANICS

TEST IV

1.) A graph of a traveling wave as seen at t = 2 seconds and t = 3 seconds is shown to the right. The wavelength of the wave is:



a.) 18 meters. [We can take the wavelength directly off the graph. One cycle spans 24 meters, which means the wavelength is 24 meters and this response is false.]

b.) 24 meters. [According to the analysis above, this is the one.]

c.) 30 meters. [Nope.]

d.) Can't tell with the information given. [Nope.]

e.) None of the above. [Nope.]

2.) The position function for an oscillating body is $x = 20 \sin(.6t - \pi/2)$. At t = 0, the body's velocity is:

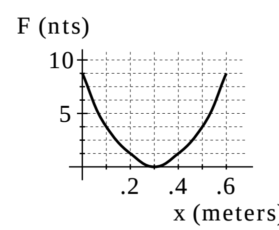
a.) 20 m/s. [There are two ways to do this: you can grunting through the math or be clever. The math yields a velocity expression equal to $v = (.6)(20)\cos(.6(0)-\pi/2) = 0$. Being clever requires you to notice that at t = 0, the displacement is at the negative maximum (i.e., $A\sin(-\pi/2) = -A$). The velocity at the extremes is zero (it is a turn around point), so the velocity here at t = 0 must be zero. In any case, this response is false.]

b.) 12 m/s. [Nope.]

c.) 2.0 m/s. [Nope.]

d.) None of the above. [This is the one.]

3.) A 4 kg body moves from x = 0 to x = .6 meters one-dimensionally over a frictionless surface. It takes 12 seconds to execute the trip. The single force acting on the body during the motion is graphed to the right. The force function in this case is $F = (x - 3)^2$.



a.) The net work done by the force over the first .3 meters is negative, whereas the work done over the final .3 meters is positive. [As the force is always in the same direction (it gets larger and smaller, but it always does so in the positive region), the work will always be positive. This statement is false.]

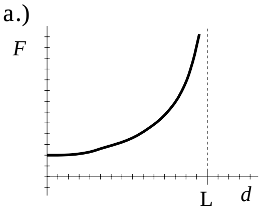
b.) The net work over the entire interval is $(x^3/3) - 3x^2 + 9x$. [This is the integral of Fdx. This is a work quantity, but it is not evaluated between x = 0 and x = .6 meters. As picky as this is going to seem, this statement is false.]

c.) The work done between x = .1 meters and x = .6 meters is 3.5 joules. [Evaluating the integral of Fdx between the limits does, indeed, give you 3.5 joules.]

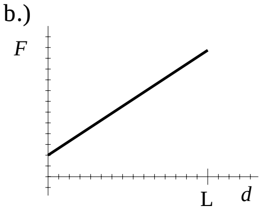
d.) None of the above. [Nope.]

4.) A ladder of mass m and length L sitting on a frictionless floor at an angle θ is perched against a frictionless wall. A horizontal force F acting at a distance d units up the ladder keeps the ladder from angularly accelerating. Which graph characterizes the relationship between F and d?

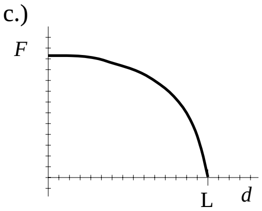
a.)



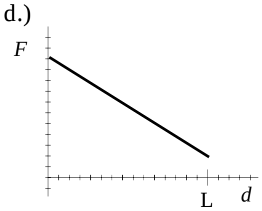
b.)



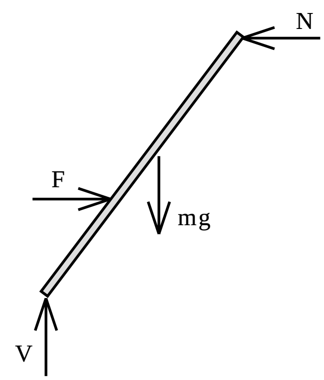
c.)



d.)



e.) None of these.



[Commentary: The temptation is to assume that the torque required of F to keep the ladder stationary must be a constant. If that is so, the larger we make d (i.e., the farther F is from the point of contact with the floor), the smaller F has to be. There is a problem with this thinking which is noticed if you actually do the problem: Using the f.b.d. shown to the right, the sum of the forces in the horizontal yields $F - N = ma_y = 0$, or $F = N$. Summing the torques about the floor yields: $-F(x \sin\theta) - mg[(L/2)\cos\theta] + NL\sin\theta = I\alpha = 0$. Substituting F for N and solving yields:

$$F = \frac{mg[(L/2)\cos\theta]}{-d\sin\theta + L\sin\theta} = \frac{mgL\cos\theta}{2(L-d)\sin\theta} = \frac{mgL\cot\theta}{2}\frac{1}{(L-d)}.$$

What does this tell us? When the force is at the floor and d = 0, the force is $\frac{mg}{2\cot\theta}$--a finite quantity. But as d approaches L (i.e., as the force gets closer and closer to the wall), the denominator gets smaller and smaller and F goes toward infinity. The graph that characterizes this kind of function is Graph a.]

5.) An ideal spring is hung free in the vertical. A 5 kg mass is attached without elongating the spring, and the mass is released from that point to oscillate up and down. If the mass had been gently lowered (it wasn't, but if it had been), it would have moved 2.5 meters downward before coming to equilibrium. Approximate g to equal 10 m/s^2.

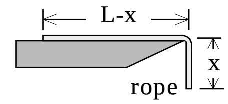
a.) The spring constant is 2 nts/meter, the angular frequency of the motion will be 2 radians/second, and the maximum velocity will be 5 m/s. [The spring constant is defined as the force per unit displacement required to stretch or compress a spring. In this case, the weight of a 5 kg mass (that is, (5 kg)(10 m/s^2) = 50 nts) elongates the spring 2.5 meters. The ratio of (50 nt)/(2.5 meters) = 20 nt/m is the force constant. As such, this response is false. The fact that the spring and mass, when gently lowered, would have reached 2.5 meters, means that the amplitude of the motion will be 2.5 meters (the amplitude is the maximum displacement from equilibrium, which is what 2.5 meters is in this problem).]

b.) The spring constant is 20 nts/meter, the angular frequency of the motion will be 4 radians/second, and the maximum velocity will be 2 m/s. [This has the correct spring constant. The angular frequency will equal $(k/m)^{1/2}$ = $[(20 \text{ nt/m})/(5 \text{ kg})]^{1/2}$ = 2 radians per second. The angular frequency here is wrong, so this response is false.]

c.) The spring constant is 2 nts/meter, and neither the angular frequency nor the maximum velocity can be calculated as there is not enough information to do so. [Wrong spring constant.]

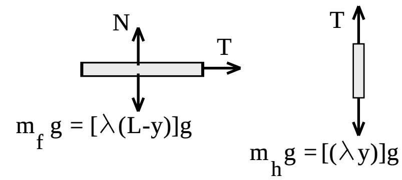
d.) The spring constant is 20 nts/meter, the angular frequency of the motion will be 2 radians/second, and the maximum velocity will be 5 m/s. [The spring constant and angular frequency are both correct and the maximum velocity will be $v_{max} = \omega A = (2 rad/sec)(2.5 m) = 5 m/s$, so this response is true.]

e.) None of the above. [Nope.]

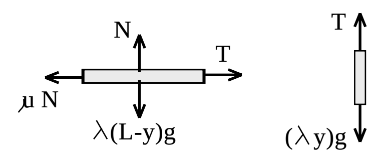


6.) At a given instant, a fat rope of length L and mass density λ (note: mass density is the mass per unit length of rope) is as shown hanging over the edge of a table.

a.) Assuming the table is frictionless, the rope's acceleration will be dependent only upon λ. [The only thing accelerating the rope is gravity pulling down on the part of the rope that is hanging over the table's edge (though there is a tension force due to the inertia of the flat-lying rope). The amount of hanging mass is equal to the mass per unit length λ times the length y of rope hanging, or $m_h = \lambda y$. By similar logic, the amount of flat-lying mass is $m_f = \lambda(L-y)$. The f.b.d.'s for the situation are shown above. N.S.L. on the hanging mass yields: $T - m_h g = m_h a$, or $T - (\lambda y)g = -(\lambda y)a$. N.S.L. on the flat-lying mass yields: $T = m_f a$, or $T - [\lambda(L-y)]a = 0$. Eliminating T yields $a = [y/(L-y)]g$. As the rope's acceleration isn't dependent upon the mass density of the rope at all, this statement is false.]



b.) Assuming the table is frictional and the rope is stationary and not accelerating, the coefficient of static friction between the rope and the table will be y/(L-y). [F.b.d.'s on both sections are shown to the right. N.S.L. applied to the tabled piece yields (after substituting $N = \lambda(L-y)g$ and putting a = 0):



$$T - \mu_k \lambda (L-y)g = 0$$

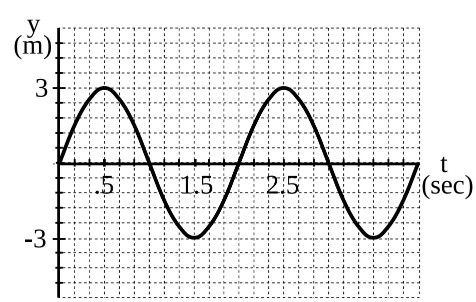
N.S.L. on the hanging piece yields:

$$T - \lambda y g = 0.$$

Substituting in for T and dividing out λ, we get $\mu_k = y/(L-y)$. This statement is true.]

c.) Back to the frictionless situation, the acceleration will be a constant. [From Part a, the acceleration will be $a = [y/(L-y)]g$. Clearly, a changes as y increases. This statement is false.]

d.) Both b and c. [Nope.]



7.) At a particular position along the path of a wave, the displacement vs. time is graph to the right.

a.) The amplitude of the wave is 3 meters, its wavelength is 2 meters, and its frequency is .5 Hz. [Looking at the graph, it is obvious that the amplitude is 3 meters. The wavelength is a quantity that is distance related, not time related. If we knew the wave velocity, which we don't, we could use the time related information on the graph to determine the frequency of the motion, then use $v = \lambda \nu$ to determine the wavelength. But because we don't have the velocity information, we don't have enough information to determine the wavelength for this situation, and this response is false.]

b.) The amplitude of the wave is 6 meters, it is impossible to tell what the wavelength is from the information given, and its frequency is 2 Hz. [Wrong amplitude--this response is false.]

c.) The amplitude of the wave is 3 meters, it is impossible to tell what the wavelength is from the information given, and its frequency is .5 Hz. [The amplitude is correct as is the observation that the wavelength can't be determined for this situation. As for the frequency, it would appear from the graph that one cycle takes 2 seconds to sweep out. That gives us a frequency of (1/2) cycles/second, which means this response is true.]

d.) None of the above. [Nope.]

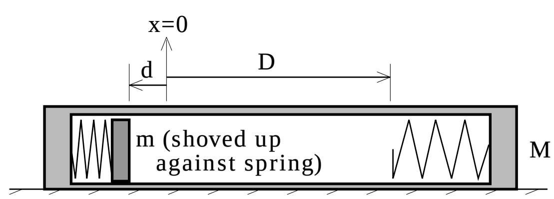
8.) A mass m is released 6R meters from the surface of a moon whose radius is R and whose mass is M. If released from rest, how fast will the mass be traveling just before it hits the surface of the planet?

a.) $[2GM/(6R)]^{1/2}$. [This is a conservation of energy problem. There are two things that can potentially be tricky about it. The first is that you have been given the distance between the object and the surface of the planet, not the object and the planet's center (the latter is what is needed for the potential energy function). The second is that the body will have gravitational potential energy when it is on the planet's surface. If you neglect to notice either of these potential pitfalls, you will write the conservation of energy equation as: KE_1 (this is zero) + U_1 + W_{ext} (this is zero) = KE_2 + U_2 (incorrectly assumed to be zero). That is, $-GmM/(6r) = .5mv^2$, concluding that $v = [-2GM/(6R)]^{1/2}$. Hopefully, you would notice the negative sign under the square root and not peremptorily change it to make the equation work. This response is not true.]

b.) $[-2GM/(7R)]^{1/2}$. [If you didn't notice that there is potential energy associated with the planet's surface, but you did get the right distance value for the potential energy function, you wrote $-GmM/(7r) = .5mv^2$, concluding that $v = [-2GM/(7R)]^{1/2}$. If you ignored the negative sign under the radical (ARGH), you got the incorrect response quoted above.]

c.) $[12GM/(7R)]^{1/2}$. [If you noticed all of the twists, you wrote: $-GmM/(7R) = .5mv^2 - GmM/R$, concluding that $v = [12GM/(7R)]^{1/2}$. This response is true.]

d.) None of the above. [Nope.]



9.) An ideal spring (spring constant k) is mounted inside a hollowed out block whose mass is M. With M held stationary, a second mass m is pushed up against the spring until the spring's displacement is d. A second spring with the same spring constant is positioned at the end of the hollow. The net distance between the ends of the two unsprung springs is D (see sketch). The shaft applies a constant frictional force f on m only over the distance marked. Assume M is fixed to the ground so that it cannot move. The spring is released accelerating m.

a.) Just after leaving the spring, m's velocity will be $(k/m)^{1/2}d$. [The spring is ideal, so energy is conserved through the acceleration. Using conservation of energy for that period, we get:

$.5kd^2 = .5mv_1{}^2$, or $v_1 = k(/m)^{1/2}d$. This response is true. Are there any other responses that are true?]

b.) The impulse provided to m by the spring will equal $(km)^{1/2}d$. [The impulse is either $F\Delta t$ or $\Delta(mv)$. We can easily determine the latter. That yields: $\Delta p = (mv_1 - mv_o) = (m\,(k/m)^{1/2}d - 0) = (km)^{1/2}d$. This statement is true. Are there others?]

c.) If m = .1 kg, D = 1 meter, f = 1 nt, and m's velocity just as it leaves the spring is 7 m/s, the time it takes m to reach the second spring will be approximately .16 seconds. [We need to use the modified conservation of energy theorem to determine the velocity of m as it reaches the second spring, then use the modified conservation of momentum equation to determine the time interval. Noting that the frictional force will do negative work in the amount -fD, conservation of energy yields: $.5m(v_{clear})^2 - fD = .5mv_2{}^2$. With numbers, this becomes $.5(.1\text{ kg})(7\text{m/s})^2 - (1\text{ nt})(1\text{ m}) = .5(.1\text{ kg})v_2{}^2$. From this, $v_2 = 5.4$ m/s. Friction is acting as an external force here (M is restrained from moving, hence the force M applies to m in the way of friction is external), so the modified conservation of momentum equation yields: $mv_{clear} - f\Delta t = mv_2$. Putting in the numbers yields $(.1\text{ kg})(7\text{ m/s}) - (1\text{ nt})\Delta t = (.1\text{kg})(5.4\text{ m/s})$, or $\Delta t = .16$ seconds. This response is true.]

d.) Both a and b but not c. [Response c is true also. This response is false.]

e.) Response a, b, and c. [This is the one.]

10.) Moving under the influence of gravity, a body's kinetic energy is KE_o. If, due to its natural motion in the gravitational field, the body's velocity changes to one-third of its original value, the change of the body's potential energy will equal:

a.) $(1/3)KE_o$. [Kinetic energy is a function of v^2. If the velocity decreases to one-third its original value, the kinetic energy decreases to $(1/3)^2KE_o$, or (1/9) of its original value. That means that 8/9 of the kinetic energy is converted to potential energy. This statement is false.]

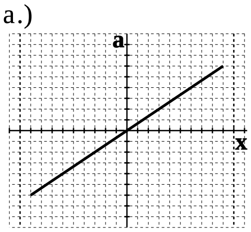
b.) $(1/9)KE_o$. [From above, this statement is false.]

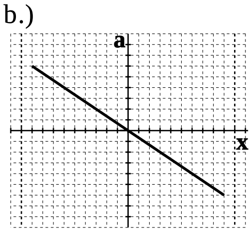
c.) $(8/9)KE_o$. [From the commentary in Response a, this statement is true.]

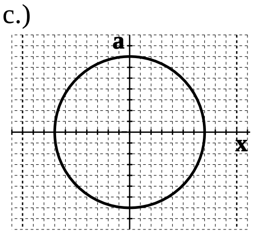
d.) $(3)KE_o$. [From above, this statement is false.]

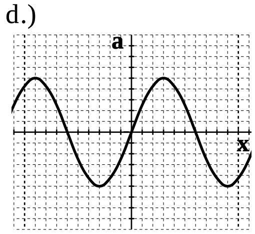
e.) $(9)KE_o$. [From above, this statement is false.]

11.) A graph of the acceleration vs. position of a body oscillating in simple harmonic motion looks like:

a.)


b.)


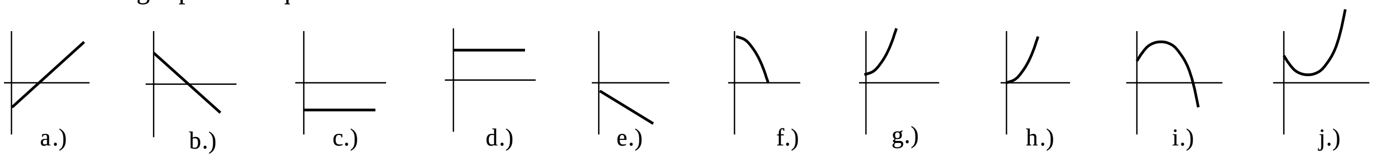
c.)


d.)


e.) None of these.

[Commentary: The position and acceleration are π radians out of phase with one another, which means that when one is big and positive, the other is big and negative. The graph that characterizes that kind of relationship is Graph b, which is the correct response for this question.]

--The graphs for questions #12 and #13:



12.) If all of the graphs were that of Position vs. Time, the only graphs listed below that would fit a kinematic situation would be:

a.) Graphs a, b, and e. [The only criteria that must be met to use kinematics is that the acceleration of the motion be a constant. Position vs. Time graphs that depict constant changes of position (graphs a, b, and e) correspond to constant velocity situations in which the accelerations are also constant and equal to zero. So, is this the right response? The answer to that hinges on whether there are other graphs in addition to these that will work. If there are, this response is not COMPLETE and won't do.]

b.) Graphs a, b, c, d, and e. [Along with graphs a, b, and e, graphs c and d also correspond to constant velocity situations (these are no-motion, constant position situations in which the velocities are a constant zero). Is this the right response? The answer to that hinges on whether there are other graphs in addition to these that will work. If there are, this response is not complete and won't do.]

c.) Graph f, g, h, i, and j. [Parabolic Position vs. Time graphs correspond to situations in which the velocity is changing in a constant manner--exactly what is needed for kinematics. So, is this the right response? Given that we have already determined that Responses a and b fit the bill, this response is incomplete and, hence, lacking.]

d.) All of the graphs. [This should now be obvious from what has been said above.]

e.) None of the graphs. [Nope.]

13.) If all of the graphs were that of Velocity vs. Time, the only graphs listed below that would fit a kinematic situation would be:

a.) Graphs a, b ,and e. [Velocities that change at a constant rate correspond to constant acceleration (i.e., kinematic) situations. Graphs a, b, and e do this, but are there others?]

b.) Graphs a, b, c, d, and e. [In addition to graphs a, b, and e, graphs c and d also depict constantly changing velocities (their changes may be zero, but they are still <u>constant</u> zeros). Are there other possibilities? The answer is NO (see the next possibility). This response is true.]

c.) Graphs f, g, h, i, and j. [The parabolic curves associated with these graphs do not represent velocity changes that are constant, which means that they do not correspond to constant acceleration situations. This response is false.]

d.) All of the graphs. [Nope.]

e.) None of the graphs. [Nope.]

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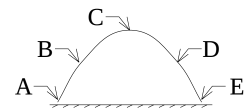


14.) A projectile is fired in a frictionless environment as shown to the right.

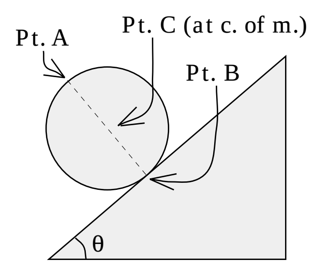
a.) The work gravity does while the body moves from Point B to Point D is zero. [Qualitatively, gravity will do equal but opposite work going up as coming down between these two points, so the net work done by gravity should be zero. From a mathematical standpoint, gravity is a conservative force. As such, the work it does as a body moves from one point to another is endpoint dependent. In the case of the gravitational potential energy function, what is important is the y coordinate of the initial and final positions (remember, U_{grav} = mgy near the earth's surface). As Points B and D are at the same y coordinate, the body's potential energy when at those two positions will be the same. As $W_{grav} = -\Delta U_{grav}$, the net work done in this case will be zero and this statement is true.]

b.) The work gravity does while the body moves in its downward arc between Point C and Point D is negative. [The force is in the negative direction and the y component of the displacement is in the negative direction, but that does not mean that the work done by the force is negative. It isn't the direction of the force alone, or the displacement alone, that determines the sign of a work quantity. What determines the sign is the direction of the two vectors relative to one another. If both are down, the angle between the two is less than 90^o and the cosine of the angle will be positive. Viewed in a different light, the force is motivating the body to pick up speed, which means the energy of the system is increasing and the work done must be positive. This statement is false.]

c.) The amount of work gravity does as the body moves from Point A to Point E is twice the amount of work done between Point A and Point C. [In fact, because the two points have the same y coordinate, the change of potential energy will be zero and the work done by gravity between those two points will be zero. Put another way, gravity does negative work on the body on its way up, and an equal amount of positive work on the way down. The net work round trip is zero. This statement is false.]

d.) Both a and b. [Nope.]

e.) Both a and c. [Nope.]



15.) A ball rolls down an incline. Its angular acceleration is:

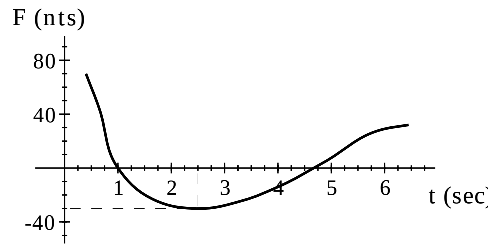
a.) mg(R $\cos\theta$)/I_{cm}, where I_{cm} is the moment of inertia about the ball's center of mass. [If torques had been summed about the point of contact between the ball and incline (Point B), and if $r_\perp$ for mg had been miscalculated to be R($\cos\theta$) instead of R($\sin\theta$), and if that torque sum was set equal to $I_{cm}\alpha$ instead of $I_B\alpha$, we would have gotten the answer given. In short, this statement is false.]

b.) fR/I_B, where f is the frictional force (it's equal to mg $\sin\theta$ - ma_{cm}) and I_B is the moment of inertia about Point B. [The frictional force produces no torque about Point B as the frictional force passes <u>through</u> Point B. This response is false.]

c.) a_A/(2R), where a_A is the acceleration of Point A. [The relationship between the ball's angular acceleration α and the translational acceleration of the ball's center of mass is a_{cm} = (1R)α. Why? Because the acceleration we are looking at is that of a point located a distance R units from the instantaneous axis of rotation of the ball at its point of contact with the incline (remember, we can evaluate the motion either by looking at the rotation about and translation of the center of mass, or from the perspective of a pure (instantaneous) rotation about the point of contact (Point B)--in this case, we are using the latter approach). Likewise, a point 2R units from the instantaneous axis of rotation (i.e., the acceleration at Point A) will have a translational acceleration $a_A = (2R)\alpha$. Rearranging this expression yields $\alpha = a_A/(2R)$. This statement is true.]

d.) Both a and c. [Nope.]

e.) Both b and c. [Nope.]



16.) A non-linear frictional force in the x direction is graphed to the right. If it is the only force acting on a body, and if the body's mass is 3 kg:

a.) At t = 2.5 seconds, the mass is moving in the -x direction. [There is no way of telling the direction of the motion given the direction of force. All the force will do is change the motion, speeding the body up or slowing it down. This response is false.]

b.) At t = 2.5 seconds, the mass's velocity will be approximately zero. [The slope of the force function is zero at t = 2.5 seconds, but all that means is that the force is not changing at that point in time. It tells us nothing about the velocity. This statement is false.]

c.) The acceleration of the mass at t = 2.5 seconds is approximately -10 m/s^2. [Using N.S.L. to evaluate the situation (approximating F to equal 30 newtons at t = 2.5 seconds) yields $F = ma \Rightarrow$ (-30 nt) = (3 kg)a, or a = -10 m/s^2. This statement is true.]

d.) The velocity of the body would either have been positive until t = 1 second, whereupon it would have changed, or vice versa. [Just because the force changes direction at t = 1 second doesn't mean the body changes directions at that point in time. This statement is false.]

e.) None of the above. [Nope.]

--The following information pertains to Problems 17 through 20: Projectiles A, B, C, and D are fired at the same time from a height h meters above the ground. With the exception of Projectile D, which is dropped from rest, all the projectiles (i.e., Projectiles A, B, and C) have the same muzzle velocity v_o, (though each is fired at a different angle--see the sketches below and note that the angle defined as θ is the same in all cases). It takes t_1 seconds for Projectile A to get to the top of its flight. It takes t_2 seconds forProjectile D to reach the ground.

Projectile A Projectile B Projectile C Projectile D

17.) The initial velocity of Projectile C will be:

a.) The same as the initial velocity of Projectile B. The two will also have the same acceleration. [They have the same acceleration and initial velocity magnitudes, but their initial velocity directions will be different.]

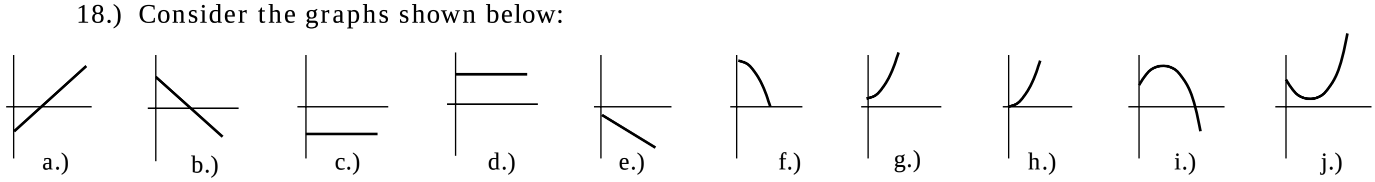
b.) The same as Projectile B after time t_1. The two will also have different accelerations. [At time t_1, Projectile B will have the same velocity as Projectile A has after time

$2t_1$. At this time, the velocity magnitude of both will be v_o at an angle θ below the horizontal. In other words, this part of the statement is true. Unfortunately, the "different accelerations" part of the statement makes it a false response.]

c.) The same as Projectile A after time $2t_1$. The two will also have the same acceleration. [From the discussion in Response b, this statement is true.]

d.) None of the above. [Nope.]

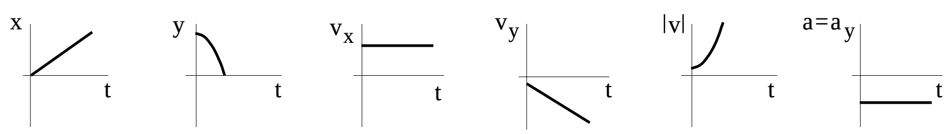
18.) Consider the graphs shown below:



Projectile C's:

a.) Y-component of Position vs. Time graph looks like graph e.
b.) X-component of Position vs. Time graph looks like graph a.
c.) Y-component of Velocity vs. Time graph looks like graph e.
d.) X-component of Velocity vs. Time graph looks like graph c.
e.) Y-component of Acceleration vs. Time graph looks like graph b.

[Commentary: A graph for each of the major parameters for this situation is shown below. This is something you should have been able to both visualize and sketch on your own. If you think you wouldn't have been able to do that, use the graphs provided as a stimulus to do the visualization part. Response c above is the one.]



--For questions 19 and 20:At the same time Projectiles A, B, and C are fired, Projectile D is dropped from rest a distance h meters above the ground. It takes this projectile t_2 seconds to reach the ground.

19.) The time t_2:

a.) Depends only on h and constant(s). [Using $x_2 = x_1 + v_1t + .5at^2$ with $a = -g$, $v_1 = 0$, and $x_2 - x_1 = 0 - (h) = -h$, we get the relationship $-h = .5(-g)t^2$. This selection is evidently true, but are there other true statements?]

b.) Is the same time it takes Projectile B to hit the ground. [The time it takes to hit the ground is a y-motion related question. As the initial velocity in the y-direction for both cases is the same (it's zero), and as the gravitational acceleration is the same in both cases, the two projectiles should hit the ground in the same amount of time. This statement is true. Other truths?]

c.) Is more than the time it takes Projectile C to hit the ground, but less than the time it takes Projectile A to hit. [Because Projectile C had a downward initial velocity in the y-direction, it will take less time to hit the ground than does Projectile D which had no initial velocity in the y-direction. And as Projectile A had an upward y-component of its velocity, it will take more time to reach the ground. This statement is true.]

d.) Both a and b, but not c.

e.) All of the above except d. [This is the one.]

20.) If h were doubled, Projectile D's:

a.) Time to touch down would double. [Using $x_2 = x_1 + v_1t + .5at^2$ with a = -g , $v_1 = 0$, and $x_2 - x_1 = 0 - (h) = -h$, we get the relationship $-h = .5(-g)t^2$. This means that $t = (2h/g)^{1/2}$. If h is doubled, t goes up by a factor of $(2)^{1/2}$, not by a factor of 2. This response is false.]

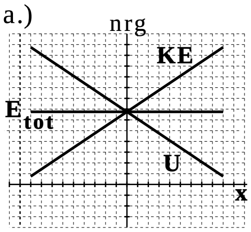
b.) Velocity just before touch down would double. [Using $v_2 = v_1 + at$ with v_2 being the velocity just before hitting the ground, $v_1 = 0$, and a = -g, we get $v_2 = -gt$. We have already determined that doubling h does not mean t doubles, so this statement if false.]

c.) Acceleration just before touch down would double. [Acceleration in these cases is always constant in both the x and y-direction. False.]

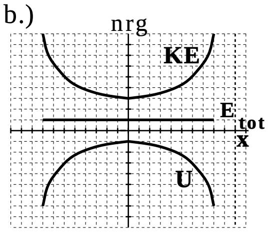
d.) None of the above. [This must be the one.]

21.) Which graph combination depicts the potential energy, the kinetic energy, and the total energy of an ideal vibrating spring?

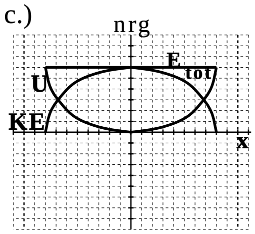
a.)



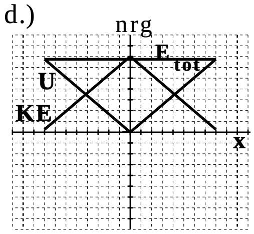
b.)



c.)



d.)



e.) None of these.

[Commentary: The kinetic energy and potential energy associated with a vibrating spring have two characteristics that will be helpful here. Both are positive ($.5mv^2$ and $.5kx^2$), which eliminates Graph b, and both must add to the constant total energy, which eliminates Graph a. We know that the potential energy function is related to the square of the displacement, so Graph d isn't going to do the job, and we can see that Graph c has all of the characteristics we are looking for. In fact, that is the correct response.]

22.) <u>In theory</u>, a mass must travel how fast to escape the earth's gravitational field?

a.) It is not possible to tell for this situation because the mass of the object was not given. [Just as the acceleration of a freely falling body is not a function of mass (due to the inertial mass/gravitational mass interplay), an object's escape velocity is also not a function of mass. This response is false.]

b.) A mass can never escape the earth's gravitational field. [In theory, it can when it reaches infinity . . . IN THEORY! This response is false.]

c.) $(2GM/R_e{}^2)^{1/2}$. [Oh, so close. Problems like this require the use of conservation of energy. As the body leaves earth, its kinetic energy is $.5m(v_{esc})^2$. Its gravitational potential energy at the earth's surface is $-GmM/R_e$. As the energy boost that initially accelerated the satellite has already happened (that's how the body got its kinetic energy), W_{ext} is zero. Once the mass has just barely escaped the earth's field, all of its energy will have been spent leaving it with no kinetic energy (i.e., $KE_2 = 0$), and as the body is theoretically at infinity, its gravitational potential energy is zero (i.e., $-GmM/\infty = 0$). In other words, the conservation of energy equation can be written as: $.5mv_{esc}^2 + (-GmM/R_e) = 0$, or $v_{esc} = (2GM/R_e)^{1/2}$, and this response just misses the mark.]

d.) None of the above. [This is the one.]

23.) When a 10 kg body is .5 meters up a 30^o incline plane, its velocity is observed to be 20 m/s. A constant 50 newton frictional force acts on the body. Over the next 4 meters (note that a 4 meter run at 30^o will produce a rise of 2 meters):

a.) The work gravity does on the body will be greater than the work friction does. Also, when the body finally gets to the top of its run, it will stop momentarily, then begin to accelerate back down the incline. [The first part of this is straightforward. Taking the zero potential energy level to be where the body has velocity 20 m/s, the work gravity does is $W_{grav} = -\Delta U = -(mgy - 0) = -(10\ kg)(10\ m/s^2)(2) = -200$ joules. The work friction does is $-fd = -(50\ nt)(4\ m) = -200$ joules. This statement is false.]

b.) The work gravity does on the body will equal the work friction does. Also, when the body finally gets to the top of its run, it will stop and remain there (i.e., it won't be accelerated down the incline). [From the analysis in Response a, the first part of this statement is true. As for the second part, the component of friction down the incline will be $mg\sin\theta = (10\ kg)(10\ m/s^2)\sin 30^o = 50$ newtons. At the top, the body will stop and kinetic friction will yield to static friction. Static friction is always greater than kinetic friction, so even if the component of gravity had been a tad larger than the kinetic frictional force, static friction would have stopped the body from accelerating down the incline. This statement is true.]

c.) The work gravity does on the body will be the same as the work friction does. Also, when the body finally gets to the top of its run, it will stop momentarily, then begin to accelerate back down the incline. [The second part of this statement is analyzed in Response b. From that, this statement is false.]

d.) The work gravity does on the body will be less than the work friction does. Also, when the body finally gets to the top of its run, it will stop and remain there (i.e., it won't be accelerated down the incline). [From the analysis in Response a, the first part of this statement is false.]

24.) Newton's Second Law is applied to a system. After a free body diagram is drawn and the forces summed, the equation $-32x^3 = 2a$ emerges, where x is the position and a is the acceleration of the body.

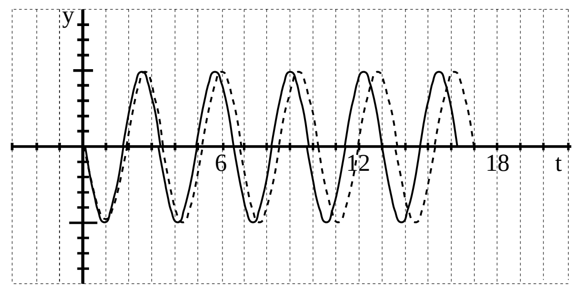
a.) This equation does not characterize an oscillatory system. [If this equation had been $-32x = 2a$, we would have had the characteristic equation for an ideal spring (i.e., $-kx = ma$). If that had been the case, a positive displacement x would yield a negative force and a negative displacement x would have yielded a positive force. That is, the force would always have been oriented back toward the equilibrium position. With the expression $-32x^3 = 2a$, we certainly don't have simple harmonic motion, but the $-x^3$ term does generate a restoring force that will motivate this system to oscillate. In short, this response is false.]

b.) This equation does characterize an oscillatory system and the motion is simple harmonic in nature. [This is an oscillatory system, but the equation is not that of simple harmonic motion. This statement is false.]

c.) This equation does characterize an oscillatory system and the motion's frequency is 4 radians per second. [As it is not simple harmonic motion, we have no way to tell what the frequency is, so this response must be false.]

d.) Both Response b and c. [Nope.]

e.) None of the above. [This is the one.]



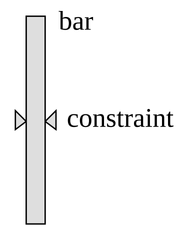
25.) Two waves exist simultaneously in the same medium (they are graphed on the same axis to the right). The beat frequency for the situation is approximately:

a.) (2/13) - (2/17). [Beats are produced when two very close frequencies superimpose. In doing so, the net waveform will sometimes be in phase and, hence, constructively interfere, and sometimes be out of phase, destructively interfering. The frequency of the warble that ensues is called the beat frequency, and the beat frequency will always equal the difference between the two superimposing waves. In this case, the frequency of the shorter wavelength is 4 cycles per 13 seconds while the frequency of the longer wavelength is 4 cycles per 17 second. The difference between these two frequencies is (4/13) - (4/17), and this response is false.]

b.) (4/13) - (4/17). [From what was said above, this is the one.]

c.) (8/13) - (8/17). [Nope.]

d.) None of the above. [Nope.]



26.) A 1.2 meter long bar is clamped at its center and tapped gently once at one end. The bar rings with a frequency equal to 200 hertz. The wave velocity of the disturbance as it moves through the bar will be:

a.) 960 m/s. [We need to determine the wavelength that will resonate on this length bar. Noting that a waveform with antinodes at both ends and a node in the middle is required, we conclude that the longest wavelength whose frequency does the job (this will be the lowest frequency to do the job--there are others) will be equal to 2L, or 2.4 meters. Using the relationship $v = \lambda \nu = (2.4m)(200hz) = 480m/s$, this response is false.]

b.) 480 m/s. [Looks like this is the one.]

c.) 240 m/s. [Nope.]

d.) None of the above. [Nope.]

27) During a 10 second time interval, the power provided to a body by a constant force is 375 watts. If the body was initially at rest:

a.) The amount of work done during the interval is 3750 joules. [Power is defined as the amount of work done per unit time, or P = W/t. Using that equation, we get (375 watts) = W/(10 sec), or W = 3750 joules. This statement is true, but it might not be the only true statement in the bunch (note that Response d says, "All of the above.")]

b.) Doubling the time over which the power-producing force is applied will quadruple the distance traveled. [How is distance related to time? The force is constant, which means the acceleration is constant. Using kinematics, the net distance d traveled (i.e., $x_2 - x_1$) is $d = v_o t$

+ $.5at^2$. With the initial velocity equal to zero, $d = .5at^2$, or the distance is proportional to the square of the time. Doubling the time should quadruple the distance, which means this statement is true. Note: This has nothing to do with power--it's essentially a kinematics problem. This is not atypical of A.P. test problems. They set you up with information given in one context (in this case, in the context of work and power), then require you to use some other approach entirely to analyze one or more of the responses. BEWARE!]

c.) Doubling the time over which the power-producing force is applied will double the power provided to the body. [From above, doubling the time quadruples the distance (i.e., the distance goes to 4d). Using the power expression, $P_{\text{time doubled}} = F(4d)/2t. = 2(Fd/t)$. This statement is true.]

d.) All of the above. [This statement is true.]

28.) A rotating disk has moment of inertia I and kinetic energy K. The body's angular momentum will be:

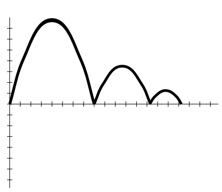
a.) K^2I. [The way to do problems like this is to write out the units for the quantity you are interested in, write out the units for the quantities you are given, then match the two. Angular momentum has the units of $I\omega$. Kinetic energy has the units of $(1/2)I\omega^2$. Multiplying the kinetic energy expression by 2I yields $I^2\omega^2$. Taking the square root yields $I\omega$. Combining the units of kinetic energy and moment of inertia to get the units of angular momentum suggests that $L = (2KI)^{1/2}$, and this response is false.]

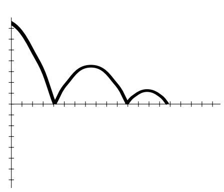
b.) $2K^2/I$. [Nope.]

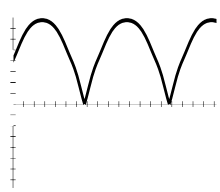
c.) $(K^2I)^{1/2}$. [Nope.]

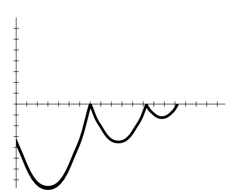
d.) $(2KI)^{1/2}$. [Yup.]

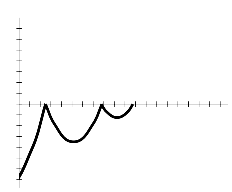
29.) A mass m is fired downward (i.e., vertically) with velocity v_o off a building that is h meters high. The mass hits the ground and bounces inelastically. Which graph shown below best depicts the body's position as a function of time?

a.)


b.)


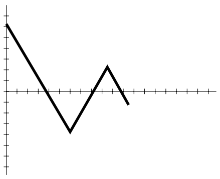
c.)


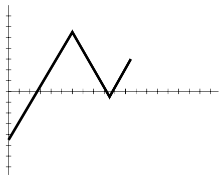
d.)


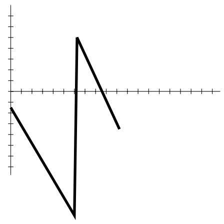
e.)


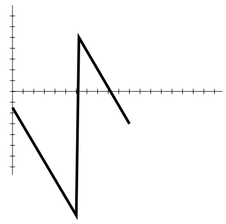
[Commentary: Visualize a ball being thrown downward. The position of the ball will always be in the positive region (i.e., above the ground), so graphs d and e cannot be right. The first motion of the ball is downward, so it can't be graphs a and c (it can't be c because energy is lost with each bounce. That leaves graph b.]

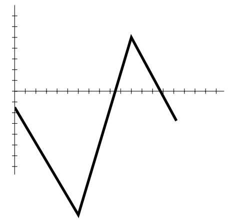
30.) A mass m is fired downward (i.e., vertically) with velocity v_o off a building that is h meters high. The mass hits the ground and bounces inelastically. Which graph shown below best depicts the body's velocity as a function of time?

a.) 

b.) 

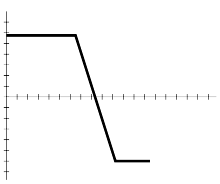
c.) 

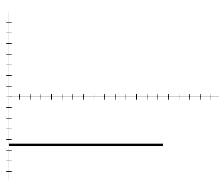
d.) 

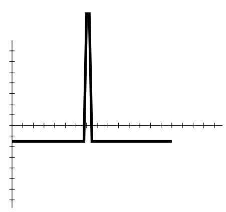
e.) 

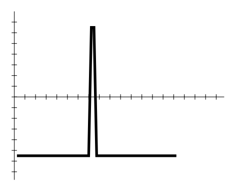
[Commentary: Again, visualize a ball being thrown downward. Its velocity will start out downward with a sign that is negative (this eliminates graph a), and its freefall velocity gets larger and larger in the negative direction until the ball hits the ground (graph b starts negative, but its magnitude gets less negative). At the ground, the velocity changes from large and negative to large and positive almost instantly. Graph e's transition from negative to positive is slow, so that's not the one. Graphs c and d appear to be the same with one small difference. The slope of the velocity on graph c after the transition is not the same as its slope before the transition. As the change of velocity (the slope) has to be the same at all times during freefall, graph d gets the nod.]

31.) A mass m is fired downward (i.e., vertically) with velocity v_o off a building that is h meters high. The mass hits the ground and bounces inelastically. Which graph shown below best depicts the body's acceleration as a function of time?

a.) 

b.) 

c.) 

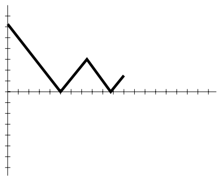
d.) 

e.) None of these graphs.

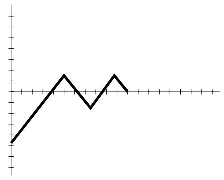
[Commentary: The mass's freefall acceleration will be negative (this eliminates graph a) and constant until the ball hits the ground. During contact with the ground, the mass's acceleration becomes positive (this eliminates graph b), very large in comparison to g (this eliminates graph d), and lasts for only an instant. Graph c fits the bill.]

32.) A mass m is fired downward (i.e., vertically) with velocity v_o off a building that is h meters high. The mass hits the ground and bounces inelastically. Which graph shown below best depicts the body's potential energy as a function of time, assuming $U_{grav} = 0$ at ground level?

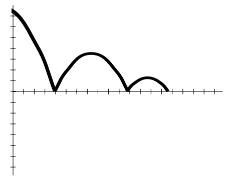
a.)



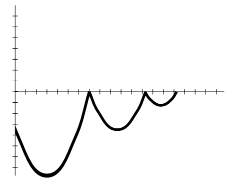
b.)



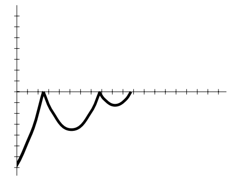
c.)



d.)



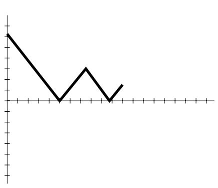
e.)



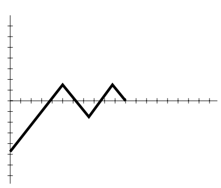
[Commentary: The gravitational potential energy function is linear with distance (mgy) near the earth's surface. Distance, on the other hand, is related to the square of the time (remember $(x_2 - x_1) = v_1 t + .5at^2$?). As such, the gravitational potential energy should vary non-linearly (this eliminates graphs a and b). As gravitational potential energy will always be positive in this case (eliminating graphs d and e), graph c is the best fit. (Notice also that after the bounce, the potential energy increases before decreasing again. Notice also that this function mimics the position vs. time function previously found).]

33.) A mass m is fired downward (i.e., vertically) with velocity v_o off a building that is h meters high. The mass hits the ground and bounces inelastically. Which graph shown below best depicts the body's kinetic energy as a function of time?

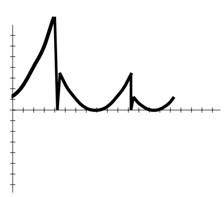
a.)



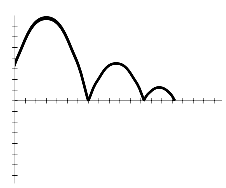
b.)



c.)



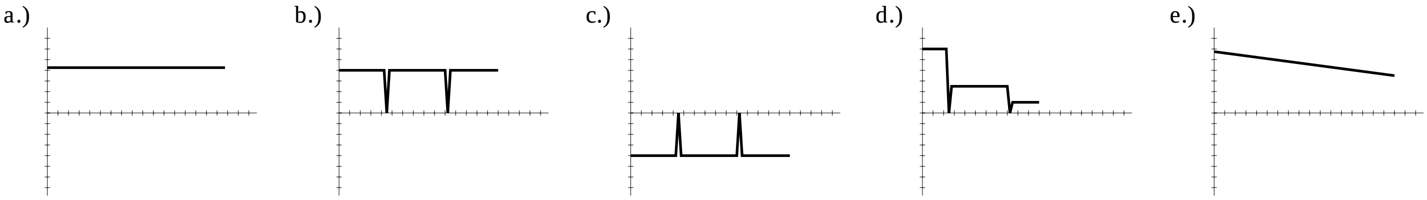
d.)



e.) None of the graphs.

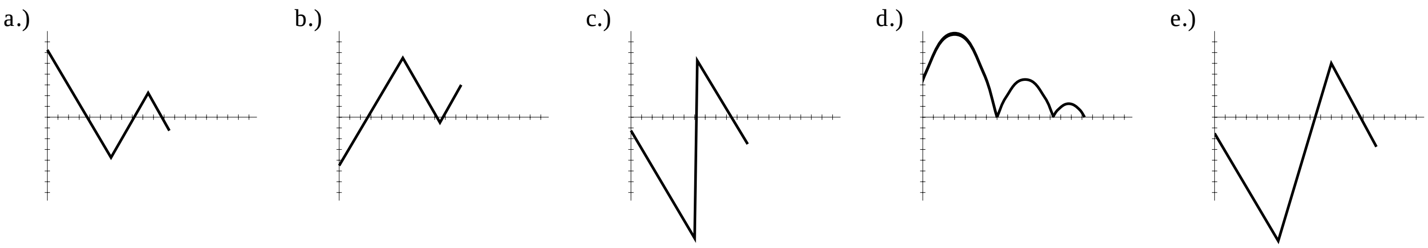
[Commentary: As potential energy is proportional to t^2, kinetic energy must also be non-linear (this eliminates graphs a and b). In fact, KE and U should be mirror images of one another (as the total energy of the system is conserved, an increase of KE must be met with an equal decrease of U, and vice versa). The mirror image to Problem 7.6c (this situation's potential energy vs time graph) is graph c. This may be difficult to see due to the long vertical lines that take KE to zero at the bounce, but if you ignore those lines you will see that the general contour of the two graphs are, indeed, inverted images of one another. If you didn't see the mirror image effect, the thing to notice is that the kinetic energy should get larger until the bounce, should go to zero at the bounce, should be very large just after the bounce, should decrease to zero as the mass rises into the air and finally stops, then should increase as it heads back toward the ground. Graph c does that. Graph d doesn't.]

34.) A mass m is fired downward (i.e., vertically) with velocity v_o off a building that is h meters high. The mass hits the ground and bounces inelastically. Which graph shown below best depicts the body's total energy as a function of time?



[Commentary: The total energy will be positive (this eliminates graph c) and conserved (i.e., constant--this eliminates graph e) as long as the mass is in the air. When it hits, the total energy (potential plus kinetic) momentarily goes to zero, then goes to a non-zero value that is less than before the bounce (this eliminates graph b). Graph d is the one.]

35.) A mass m is fired downward (i.e., vertically) with velocity v_o off a building that is h meters high. The mass hits the ground and bounces inelastically. Which graph shown below best depicts the body's momentum as a function of time?



[Commentary: Think again about the mass and its initial motion. Its velocity is negative, which means its momentum will be negative (this eliminates graphs a and d). The velocity magnitude gets bigger as the mass falls, hence momentum magnitude will also get bigger in a negative sense (this eliminates graph b). The velocity changes from a big negative value to a relatively big positive value almost instantaneously (i.e., through the bounce), hence the momentum does the same (this quick change eliminates graph e). Graph c looks like the one.]